

Chaos and Universality in a Four-Dimensional Spin Glass

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We present a finite size scaling analysis of Monte Carlo simulation results on a four dimensional Ising spin glass. We study chaos with both coupling and temperature perturbations, and find the same chaos exponent in each case. Chaos is investigated both at the critical temperature and below where it seems to be more efficient (larger exponent). Dimension four seems to be above the critical dimension where chaos with temperature is no more present in the critical region. Our results are consistent with the Gaussian and bimodal coupling distributions being in the same universality class.

I. INTRODUCTION

While the nature of the spin glass phase is still controversial, there is a property, namely static chaos, that emerges as a common feature of different spin glass models. This chaotic behavior has been studied within mean-field theory¹⁻³, scaling⁴ or droplet⁵ theories and using a real space renormalisation-group approach^{6,7}. Chaos means that the frozen random equilibrium state of the spin glass phase is completely reorganised by a small change in an external parameter, such as temperature (T) or magnetic field. Chaos with respect to a slight change in the the couplings, a so-called random perturbation, has also been studied. In the case of a temperature preturbation, for example, the spin-spin correlation varies chaotically with T, the larger the distance between the spins, the bigger this effect. The temperature scale at which the correlation function varies is

$$\Delta T_L \sim L^{-\zeta}, \quad (1)$$

which defines the Lyapunov or chaos exponent ζ . The existence of such a scaling and of ζ has been shown within the models quoted above and a summary of the findings will be presented later in this introduction. Here we study chaos by Monte Carlo (MC) simulations of a four-dimensional spin glass, and we consider the Edwards-Anderson model with Gaussian coupling distribution. Some of the questions adresssed in this paper are:

- Are all perturbations equivalent ? Chaos with two different random perturbations and a temperature change have been studied at the critical temperature and are found to give a unique exponent, within the uncertainties. The amplitudes of the effect are not the same, however, and a temperature perturbation is more difficult to see numerically than the random perturbations.
- Are there different chaos exponents at the critical temperature and in the spin glass phase ? We get rather different values of ζ , at T_c and below. This seems to support previous findings that chaos is more effective in the spin glass phase (larger exponent)⁷.

A brief reminder is needed to compare these results with ones of other models. Chaos in spin glass phase was first pointed out in mean-field theory by Parisi¹, and later confirmed³. A loop-expansion around Parisi's solution for dimensions $d \geq 8$ allowed Kondor et al.² to show chaos with magnetic field and also with temperature. Two distinct exponents ζ resulted while the temperature perturbation has a smaller effect.

For low dimensional systems, chaos has been extensively studied within the scaling theory of Bray and Moore⁴ and the droplet theory of Fisher and Huse⁵ which are based on a real space renormalisation group approach. The latter permits one to determine chaos exponents for arbitrary perturbations and in various points of the phase diagram. Chaos has been shown to be characteristic of each fixed point of the diagram⁷. In particular, one gets different exponents at the critical temperature (T_c) (between spin glass and paramagnetic phases) and in the ordered phase ($T < T_c$), in dimension three. One can argue that there is a critical dimension above which there is *no* chaos with temperature anymore at T_c ⁷. In the Migdal-Kadanoff (MK) framework which is the renormalisation scheme that has been mainly used, dimension four is above this critical dimension. This leads to an additional question for our study:

- What is this critical dimension above which chaos with temperature no longer exists at the critical temperature? Our Monte Carlo simulation indicates that four is close to but higher than this critical dimension.

Moreover, within MK, the exponents from a temperature perturbation or a random perturbation have to be the same, keeping everything else fixed, (in agreement with this paper's results). Finally, a magnetic field perturbation has been less studied but seems to lead to a different exponent⁸. As it was the case with mean-field approach, chaos with temperature is less effective (smaller exponent) than with magnetic field. A major difference, however, is that a magnetic field destroys the spin glass phase and chaos is studied in the paramagnetic phase close to zero field while, in mean field, one stays below a non-zero critical field.

MC simulations of the Edwards-Anderson model allow comparisons with both previous approaches⁹. Ritort's simulations¹⁰ of the mean-field version of this model confirms the analytical results of Kondor. In dimension two, where T_c is zero, we have been able to observe chaos with both temperature and random perturbation and found a unique exponent¹⁴. We are now interested in higher dimensions where there is a spin glass phase. Since dimension four has a clear transition in contrast to dimension three¹¹, and also because there has already been some studies of chaos for $\pm J$ random couplings¹², we focus on a Gaussian coupling four-dimensional system. Thus we are able to answer to the question :

- Are exponents the same for different coupling distributions ? The results are consistent with the Gaussian (this paper) and bimodal (Ritort et al.¹²) distributions lying in the same universality class.

Chaos with temperature or magnetic field are believed to explain cycling temperature¹⁵ or magnetic field¹⁶ experiments on spin glasses. Such an experiment has been done recently on a disordered ferromagnet¹⁷. This can be understood within renormalisation group approach⁷ where chaos is shown to be present when the coupling distribution is shifted towards ferromagnetic couplings. It could be interesting to check that chaos can also be seen with Monte Carlo simulations.

II. THE MODEL

The Hamiltonian (for Ising spins $\{S_i\}$ and nearest neighbor couplings $\{J_{ij}\}$), is

$$\mathcal{H} = - \sum_{\langle i,j \rangle} J_{ij} S_i S_j , \quad (2)$$

where the couplings are drawn from a Gaussian distribution with zero mean and variance $\overline{J_{ij}^2}$ equal to unity. The spins lie on a four-dimensional cubic lattice of linear size L with periodic boundary conditions. As in previous studies¹¹⁻¹⁴, the basic quantity is the replica overlap between two copies (replicas a and b) of the system

$$q_{ab} = \frac{1}{L^2} \sum_{i=1}^{L^2} S_i^{(a)} S_i^{(b)} . \quad (3)$$

From this the Binder ratio is computed,

$$g \equiv \frac{1}{2} \left[3 - \frac{\langle q^4 \rangle}{\langle q^2 \rangle^2} \right] \quad (4)$$

where $\langle \dots \rangle$ denotes both the average over disorder and the statistical mechanics (Monte Carlo) average. It is dimensionless and has a finite size scaling which allows for calculation of the critical temperature

$$g = \tilde{g} \left(L^{1/\nu} (T - T_c) \right) . \quad (5)$$

A similar approach has been introduced by Ritort¹² to investigate chaos with random perturbation. One has now to compare two copies with correlated coupling sets. We obtain the perturbed couplings $\{J'_{ij}\}$ from the unperturbed ones $\{J_{ij}\}$ in two ways. First,

$$(1) \quad J'_{ij} = \frac{J_{ij} + x_{ij} \Delta J}{\sqrt{1 + \Delta J^2}} , \quad (6)$$

where x_{ij} is a Gaussian random variable with zero mean and unit variance. Secondly, we consider the perturbation that changes the sign of a small fraction of the couplings,

$$(2) \quad J'_{ij} = -J_{ij} \quad \text{with probability } p \quad (7)$$

and $J'_{ij} = J_{ij}$ otherwise. With $\pm J$ couplings, one can only study case (2)¹². In order to compare both perturbations, one can determine how the two sets of couplings are correlated. One gets

$$\begin{aligned} (1) \quad \overline{J_{ij} J'_{ij}} &\simeq 1 - \frac{\Delta J^2}{2} \\ (2) \quad \overline{J_{ij} J'_{ij}} &= 1 - 2p . \end{aligned} \quad (8)$$

where (1) is expanded for small ΔJ .

A measure of the spin reorganisation under small perturbation is the chaos parameter^{12,14}

$$r_{\Delta J} \equiv \frac{\langle q_{JJ'}^2 \rangle}{\langle q_{JJ}^2 \rangle}, \quad (9)$$

where $q_{JJ'}$ is given by Eq.(3) with two copies a and b having now slightly different couplings. Note that $q_{JJ} = q_{JJ'}$ since $\{J'_{ij}\}$, and the $\{J_{ij}\}$ have the same distribution. The scaling of this quantity, at fixed temperature, leads to a chaos exponent for each random perturbation (1) and (2),

$$r_{\Delta J} = \tilde{r}_{\Delta J} (L^\zeta \Delta J) \quad (10)$$

For a temperature perturbation, one can follow the same pathway¹⁴ and define a parameter $r_{\Delta T}$ like in Eq. (9)

$$r_{\Delta T} \equiv \frac{\langle q_{T_+T_-}^2 \rangle}{\sqrt{\langle q_{T_+T_+}^2 \rangle \langle q_{T_-T_-}^2 \rangle}}, \quad (11)$$

where $q_{T_+T_-}$ is given by (3) with two copies a and b having slightly different temperatures, T_- and T_+ , which are equally shifted from a reference temperature T , that is,

$$T_\pm = T \pm \Delta T/2. \quad (12)$$

Finally, an analogous finite size ansatz is applied to this parameter

$$r_{\Delta T} = \tilde{r}_{\Delta T} (L^\zeta \Delta T), \quad (13)$$

which could lead to another chaos exponent ζ (although we use the same symbol).

III. RESULTS AND DISCUSSION

A. Chaos at T_c

A plot of g as a function of T for various sizes L shows an intersection at $T_c = 1.8$, see Fig.1. Knowing this permits one to determine the critical exponent ν from a plot of g as a function of $(T - T_c)L^{1/\nu}$, as usual^{13,11}. In such a plot, see Fig.2, a collapse of all data to a single curve is obtained for

$$T_c = 1.8 \pm 0.05, \quad \nu = 0.87 \pm 0.15. \quad (14)$$

This is in agreement with previous MC simulations^{9,13}.

Scaling plots of the chaos parameter for the three perturbations are shown in Fig. 3, 4 and 5 where the reference temperature is precisely T_c . In the case with random perturbations, Eqs. (6, 7), and temperature change given by Eq. (12) with $T = T_c$, one gets

$$\begin{aligned} \text{chaos with } \Delta J : \zeta &= 0.85 \pm 0.1 \\ \text{chaos with } \Delta T : \zeta &= 0.95 \pm 0.2. \end{aligned} \quad (15)$$

One can check that exponents from both random perturbations are equal (see Fig. 3 and 4). The data for chaos with T in Fig.5 has larger statistical errors and also deviates less from unity; in other words, the amplitude of chaos with T is smaller which is also the case with other simulations¹² and mean-field calculations².

We conclude that the exponents for chaos with ΔJ and ΔT at T_c , given by Eq. (15), are equal within the error bars. We use this to compare results from several sources for chaos with ΔJ and ΔT together in Table I.

In order to define chaos with ΔT in the critical region it is necessary that $\zeta > 1/\nu$ so that the typical temperature interval on which the spin correlation varies, $\Delta T_L \sim L^{-\zeta}$, is smaller than the critical temperature range given by $|T - T_c| \sim L^{-1/\nu}$. Both exponents depends on dimension. In our four dimensional system $1/\nu = 1.15$, and the inequality does not seem to be satisfied (or there might be an equality) ruling out chaos with T in the critical region. This is also the case with MKRG which gives $1/\nu = 0.68$ (to be compared with $\zeta = 0.53$ in Table I). According to MKRG, chaos at the critical temperature *is* present in dimension three, $1/\nu = 0.35$; this is not yet confirmed by Monte Carlo simulations.

Finally, one can see from Table I, in dimension four (numbers in brackets), that the exponents for Gaussian and $\pm J$ distributions are equal (error bars are of order ± 0.1).

TABLE I. Values of the chaos exponent is shown for various dimensions. In some cases we have ζ both in the spin glass phase, $T < T_c$, and in the critical region, at T_c (numbers in brackets). The models are Migdal Kadanoff Renormalisation group calculations (MKRG)⁷, mean-field expansions around Parisi's solution, Monte Carlo simulations for Gaussian (G) coupling distributions (this paper's results in dimension four, and our previous results in dimension two¹⁴), and bimodal ($\pm J$) coupling distribution (Note that Ritort et al. used another symbol $\lambda \equiv 2/\zeta$) and finally exact ground state numerical calculations. The chaos exponent has been determined both with random and temperature perturbations except when it is mentioned ΔJ or ΔT .

dimension \rightarrow model \downarrow	2 $T_c = 0$	3 $T < T_c(T_c)$	4 $T < T_c(T_c)$	≥ 8 $T < T_c$
MKRG ⁷	0.73	0.73 (0.57)	0.73 (0.53)	-
Mean-field ^{2,3} (ΔT)	-	-	-	1.0
Monte Carlo ¹² ($\pm J$)	-	-	1.0 (0.75)	-
Monte Carlo (G)	1.0	-	1.2 (0.85)	-
Ground State ¹⁸ (ΔJ)	0.95	-	-	-

B. Chaos in the spin glass phase

We also calculate the chaos exponent in the spin glass phase. From a previous numerical result of Young et al.¹⁹, temperature $T = 1.4$ and sizes $L \geq 3$ seems to probe the ordered low temperature phase. The data for $T = 1.4$ is shown in Fig. 6 from which we estimate

$$\text{chaos with } \Delta J, \text{ case (1)} : \quad \zeta = 1.2 \pm 0.1 . \quad (16)$$

This exponent is larger (although we cannot rule out an equality) than the one at T_c . This indicates that chaos is more efficient at low temperature in agreement with other sources quoted in Table I, see the dimension four column.

C. Conclusions

We find that the Gaussian and bimodal distributions seems to be in the same universality class (in contrast to numerical calculations of other critical exponents²⁰).

Our results on chaos in four dimensions (this paper) and in two dimensions¹⁴ are in qualitative agreement with those of the real-space-renormalisation-group approach⁷. Some of the results obtained in this approach⁷ and observed numerically are the following. First, chaos is also present in the paramagnetic phase (e.g., in dimension two¹⁴). Second, a temperature perturbation generates a random perturbation and thus gives the same chaos exponent (confirmed numerically in both two and four dimensions). Moreover, chaos with temperature occurs inside the critical region if the dimension is smaller than a limit dimension⁷, called d_+ , with $3 < d_+ < 4$ (in agreement with this paper's results). The chaos exponent is smaller at T_c than in the spin glass phase (see Table I). More generally, it has been argued⁷ that a chaos exponent can be assigned to each fixed point of each random system, and can appear in physical quantities when a perturbation of the thermodynamic parameters couples to a random perturbation. The disordered ferromagnet is one example¹⁷, and it could be interesting to perform a Monte Carlo simulation of a spin glass with random ferromagnetic couplings.

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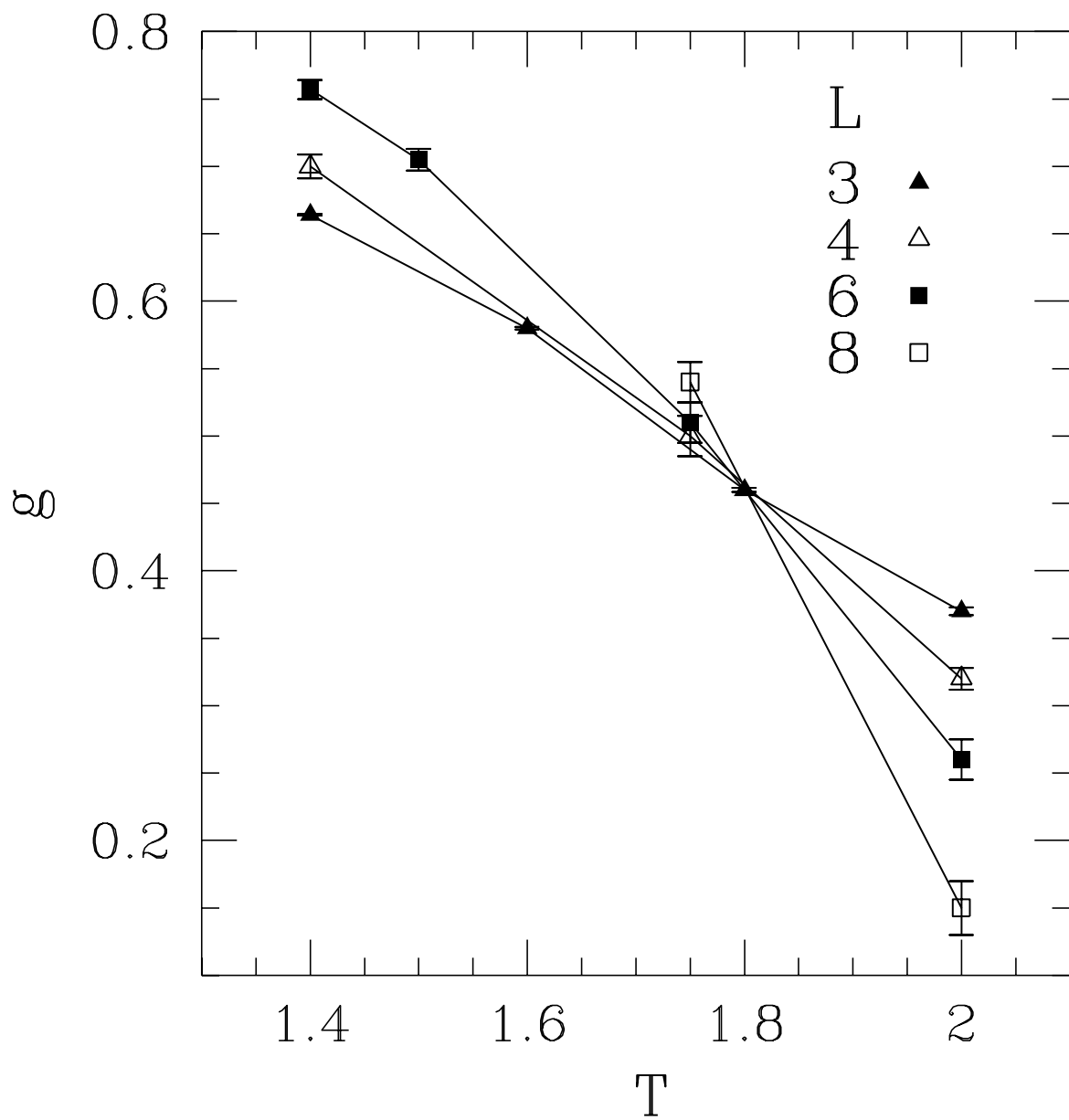


FIG. 1. Data for g against T . From the intersection, T_c is estimated to be 1.8.

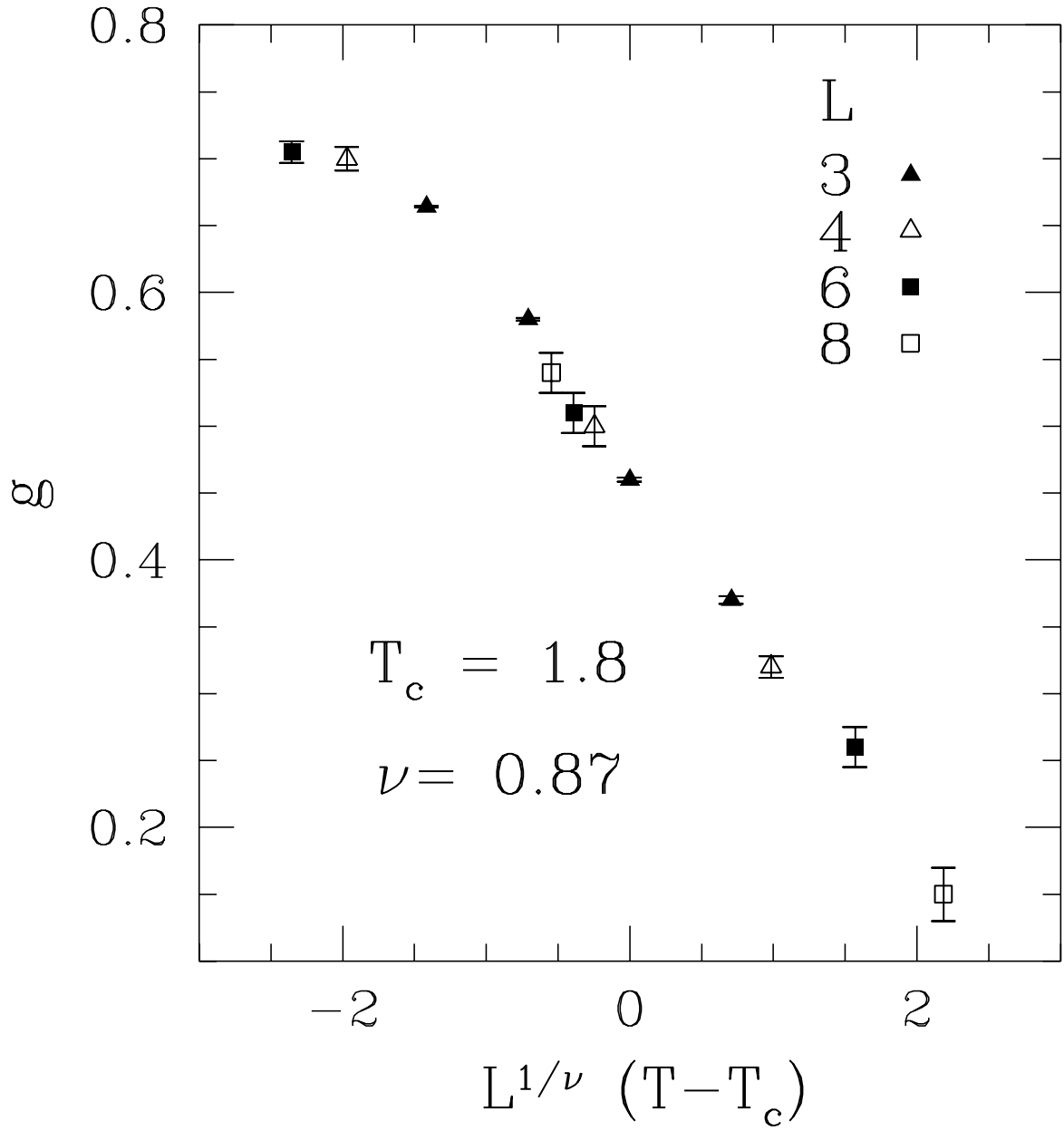


FIG. 2. A scaling plot of the data in Fig.1 scaled according to Eq. (5) which gives $\nu = 0.87$ ($1/\nu = 1.15$).

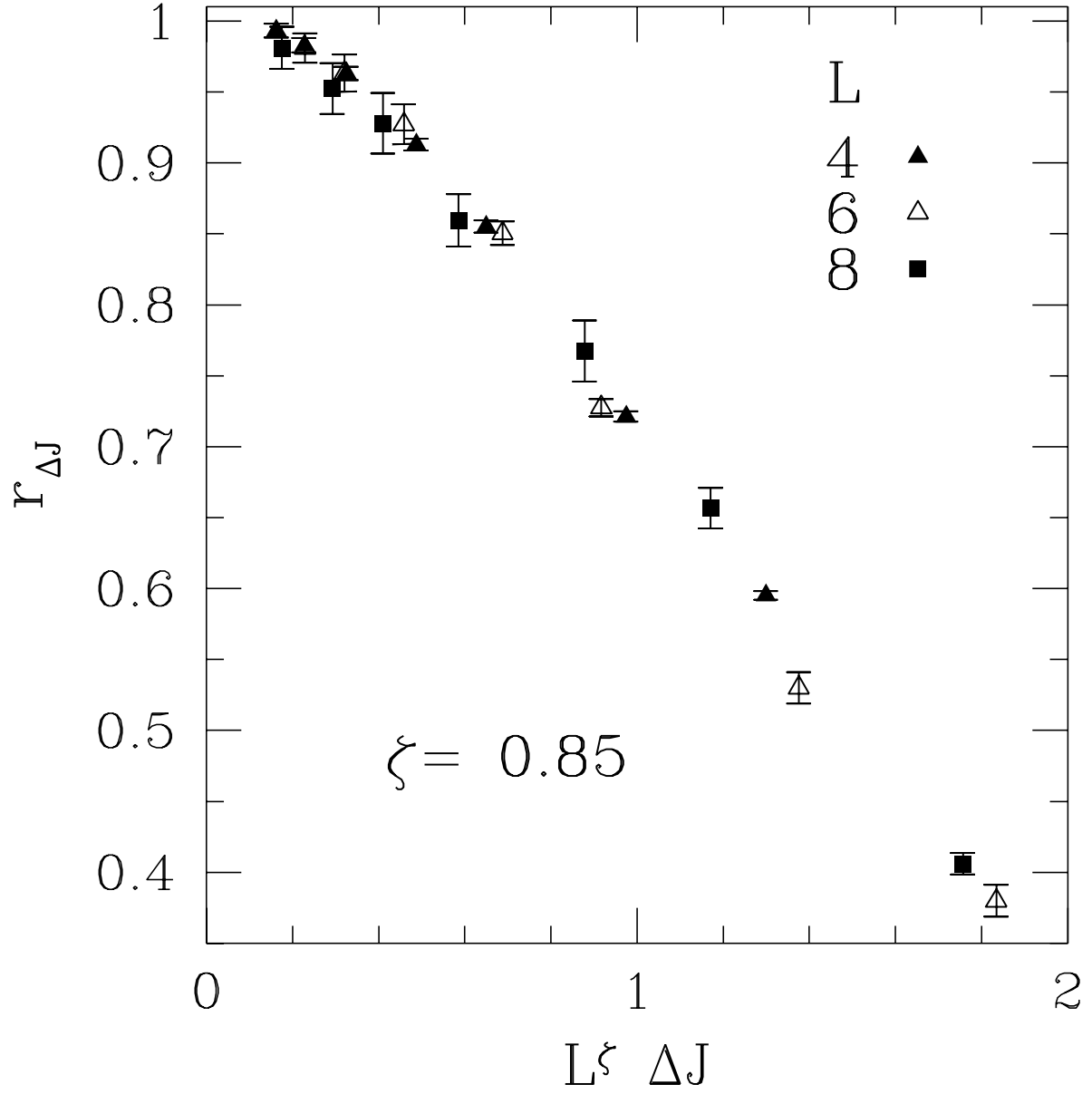


FIG. 3. A scaling plot of $r_{\Delta J}$ with random perturbation, case (1), at T_c . The perturbation lies in the range $0.05 - 0.4$. Trying different values, our best estimate is $\zeta = 0.85$.

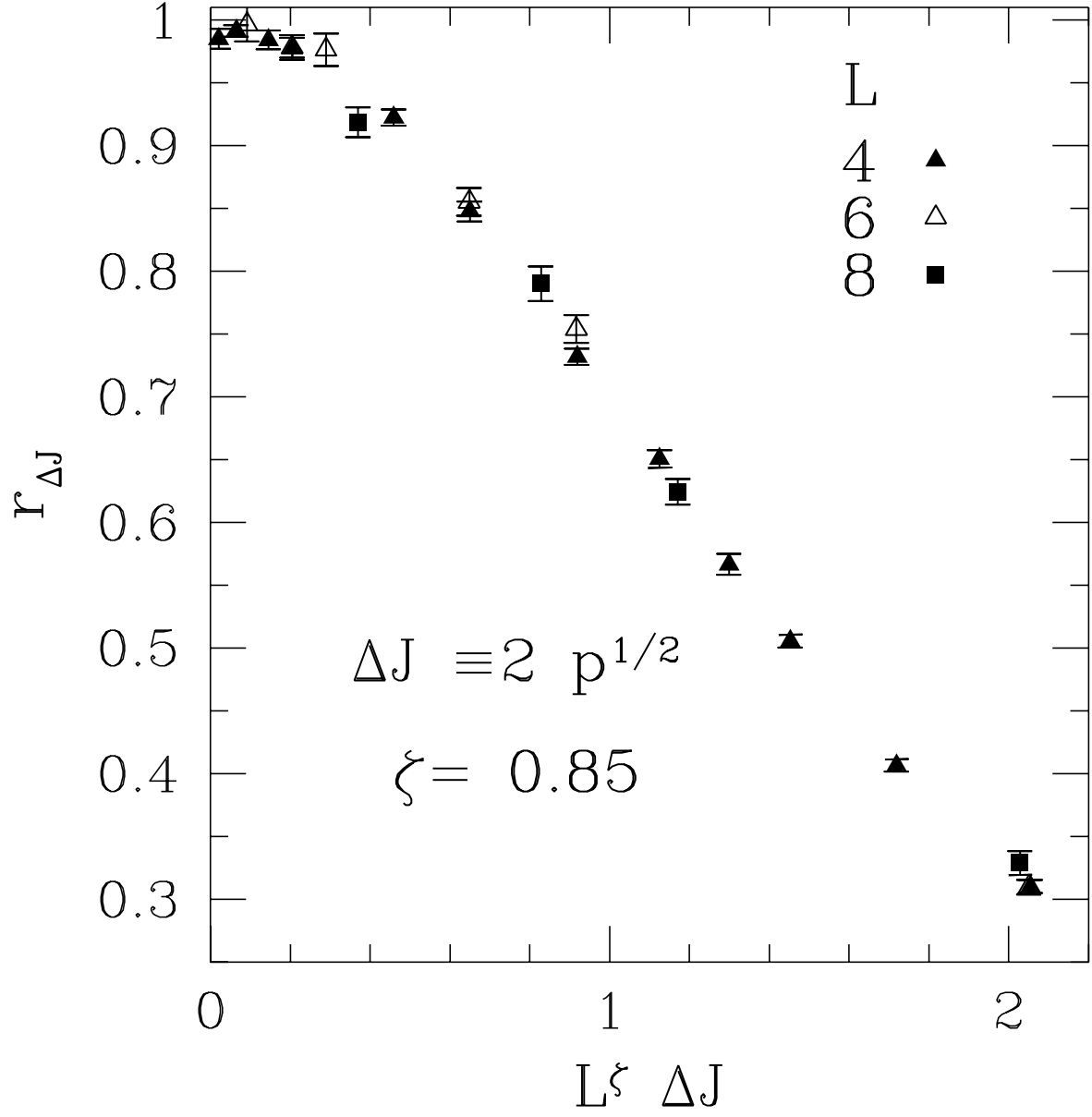


FIG. 4. A scaling plot of $r_{\Delta J}$ with random perturbation, case (2), at T_c . To compare these data with Fig. 3, we use Eq. (8) and define $\Delta J \equiv 2p^{1/2}$. The perturbation p lies in the range $0.001 - 0.1$ which is $\Delta J = 0.063 - 0.63$. The chaos exponent is $\zeta = 0.85$.

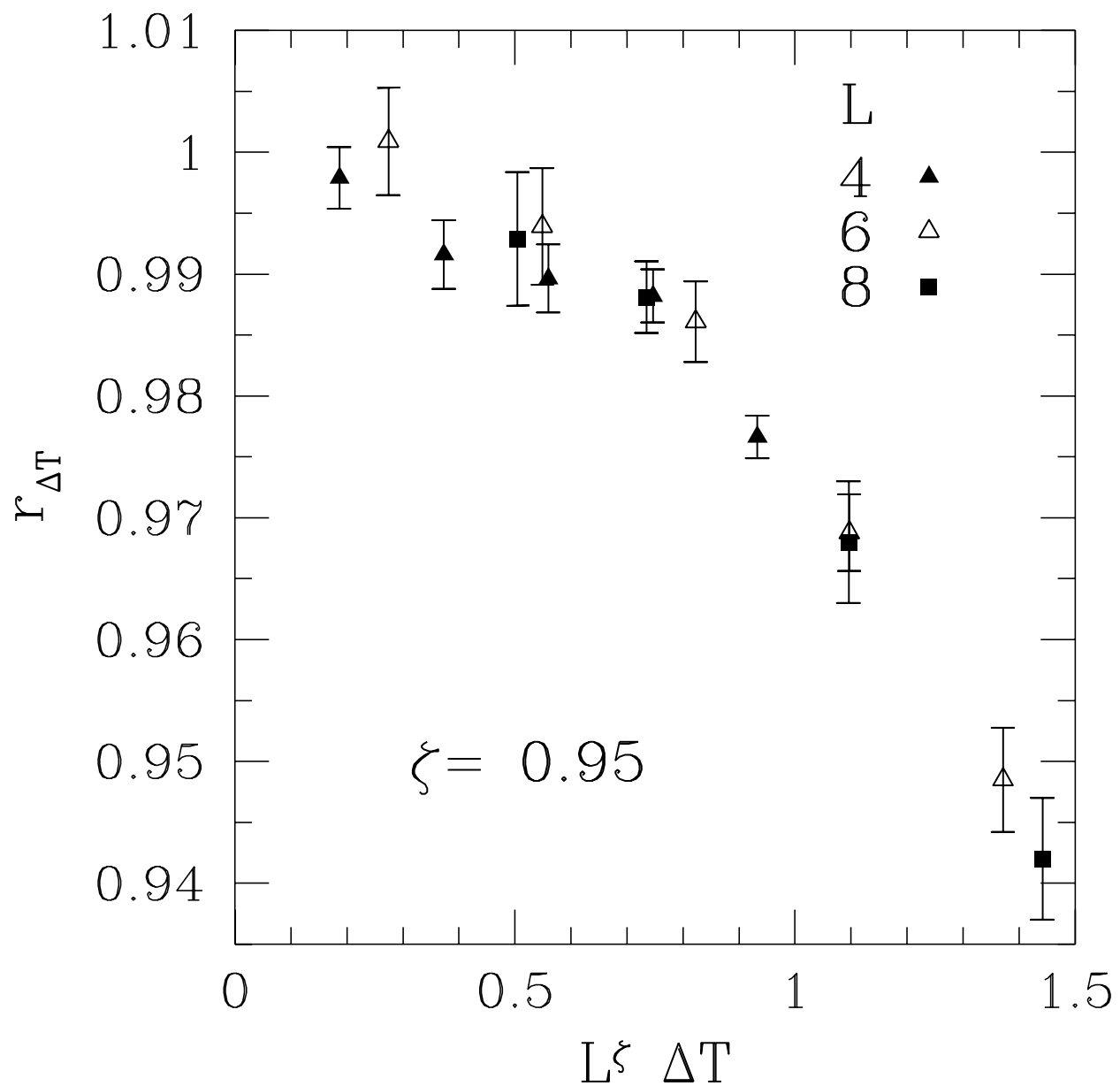


FIG. 5. A scaling plot of $r_{\Delta T}$ at T_c . The perturbation, ΔT , lies in the range $0.05 - 0.25$. The chaos exponent is $\zeta = 1.0$.

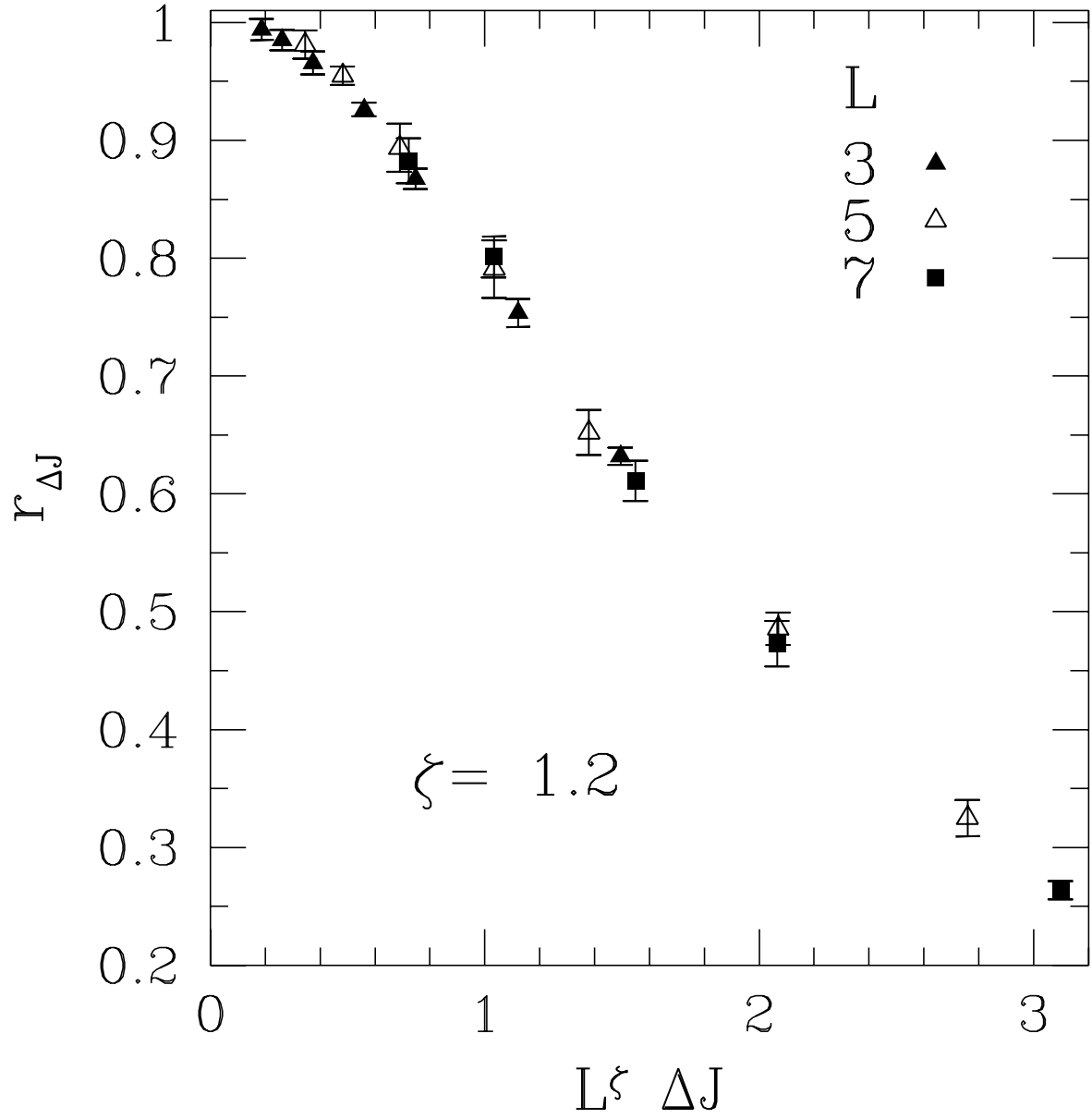


FIG. 6. A scaling plot of $r_{\Delta J}$ with random perturbation, case (1), at $T = 1.4$ in the spin glass phase. The perturbation lies in the range $0.05 - 0.4$. The chaos exponent is $\zeta = 1.2$.